Midterm 2: Math 20D Practice Final Exam
Instructor: Finley McGlade

You have 3 hours.

You are permitted the use of a **scientific** calculator and a double sided page of handwritten notes.

UNLESS THE QUESTION SPECIFICALLY SAYS OTHERWISE YOU MUST SHOW ALL YOUR WORKING.

Name _____

PID _____

"I have adhered to UCSD policies on academic integrity while completing this examination."

Signature _____

The exams consists of 10 pages (including the cover page) with four questions. The maximum possible score is **190 points**. The first thing you should do when writing time begins is CHECK to make sure you have all 10 of the pages.

Good luck!

Scratch Working Page

Problem 1. *(30 Points) In parts (a) and (b) below we make reference to the equation*

$$\frac{dy}{dt} = 2t\cos^2(y). \tag{1}$$

(a) (20 points) Using separation of variable solve (1) subject to the initial condition $y(0) = \pi/4$.

(b) (10 points) Solve (1) subject to the initial condition $y(0) = \pi/2$.

Problem 2. (30 points)

(a) (15 points) Using the method of integrating factors, solve the intial value problem

$$\frac{dy}{dx} + 4y - e^{-x} = 0, \qquad y(0) = \frac{4}{3}.$$

(b) (15 points) Solve the initial value problem in (a) via the method of Laplace transform.

Problem 3. (30 points)

(a) (20 points) Using the method of variation of parameters, find a particular solution to the equation

$$y'' + 2y' + y = e^t.$$

(b) (10 points) Write down a general solution to the equation in (a).

Problem 4. *(30 points) Using the method of Laplace transform solve the initial value problem*

$$y'' + 2y' + 2y = \delta(t) + u(t-2), \quad y(0) = 0, \quad y'(0) = 0.$$

Problem 5. *(30 points) Find a general solution to the equation*

$$\mathbf{x}'(t) = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \mathbf{x}(t)$$

where $\mathbf{x}(t) = \mathrm{col}(x_1(t), x_2(t))$.

Problem 6. (40 points) *A 1/4-kg mass is attached to a spring with stiffness coefficient 8 N/m. The damping constant for the system is 2N-sec/m. The mass is pushed 50cm left of the equilibrium and given a leftward velocity of 2m/sec. Determine the maximum leftward displacement attained by the mass.*

Scratch Working Page

A TABLE OF LAPLACE TRANSFORMS

	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1.	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$	**20.**	$\frac{1}{\sqrt{t}}$	$\frac{\sqrt{\pi}}{\sqrt{s}}$
2.	$e^{at}f(t)$	$F(s-a)$	**21.**	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
3.	$f'(t)$	$sF(s)-f(0)$	**22.**	$t^{n-(1/2)}, \quad n = 1, 2, \ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+(1/2)}}$
4.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)$	**23.**	$t^r, \quad r > -1$	$\frac{\Gamma(r+1)}{s^{r+1}}$
		$-\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$	**24.**	$\sin bt$	$\frac{b}{s^2+b^2}$
5.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$	**25.**	$\cos bt$	$\frac{s}{s^2+b^2}$
6.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$	**26.**	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
7.	$\int_0^t f(v)dv$	$\frac{F(s)}{s}$	**27.**	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
8.	$(f*g)(t)$	$F(s)G(s)$	**28.**	$\sinh bt$	$\frac{b}{s^2-b^2}$
9.	$f(t+T) = f(t)$	$\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-sT}}$	**29.**	$\cosh bt$	$\frac{s}{s^2-b^2}$
10.	$f(t-a)u(t-a), a \geq 0$	$e^{-as}F(s)$	**30.**	$\sin bt - bt\cos bt$	$\frac{2b^3}{(s^2+b^2)^2}$
11.	$g(t)u(t-a), \quad a \geq 0$	$e^{-as}\mathcal{L}\{g(t+a)\}(s)$	**31.**	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
12.	$u(t-a), \quad a \geq 0$	$\frac{e^{-as}}{s}$	**32.**	$\sin bt + bt\cos bt$	$\frac{2bs^2}{(s^2+b^2)^2}$
13.	$\prod_{a,b}(t), \quad 0 < a < b$	$\frac{e^{-sa}-e^{-sb}}{s}$	**33.**	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
14.	$\delta(t-a), \quad a \geq 0$	e^{-as}	**34.**	$\sin bt\cosh bt - \cos bt\sinh bt$	$\frac{4b^3}{s^4+4b^4}$
15.	e^{at}	$\frac{1}{s-a}$	**35.**	$\sin bt\sinh bt$	$\frac{2b^2s}{s^4+4b^4}$
16.	$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$	**36.**	$\sinh bt - \sin bt$	$\frac{2b^3}{s^4-b^4}$
17.	$e^{at}t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}$	**37.**	$\cosh bt - \cos bt$	$\frac{2b^2s}{s^4-b^4}$
18.	$e^{at}-e^{bt}$	$\frac{(a-b)}{(s-a)(s-b)}$	**38.**	$J_v(bt), v > -1$	$\frac{\left(\sqrt{s^2+b^2}-s\right)^v}{b^v\sqrt{s^2+b^2}}$
19.	$ae^{at}-be^{bt}$	$\frac{(a-b)s}{(s-a)(s-b)}$			